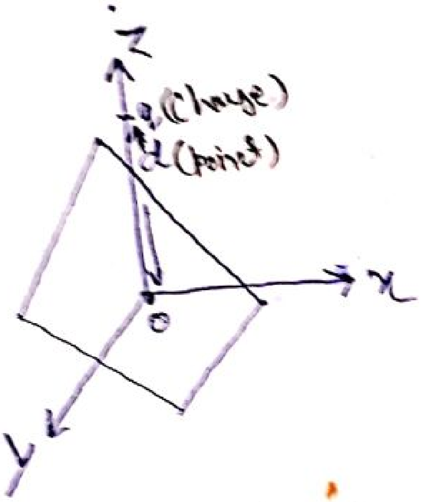
Case (I):- Point charge near an infinited grounded Conductor — (grounded Conducting plane)

~~Set~~



Suppose, a point charge q held at a distance d ~~above~~ ~~an~~ grounded infinite Conducting plane. (at surface V=0 (z=0))

what is potential in the region above the plane?

It's not just

$$\phi(x) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(x')\,d^3x'}{|\underline{x}-\underline{x}'|} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

bcoz q will induced by a certain amount of (−) charge on the nearby surface of the Conductor. the total potential ~~now, will be~~ is due ~~to~~ in part to q directly and in part to this induced charge we Cannot Calculate the potential. and How much charge is induced or how it is distributed?

from a Mathematical point of view our prob. is to Solve Poisson's eqn in the Region $z>0$, with a single point charge q at (0,0,d), subject to the boundary Conds:-

(i) $\phi(0,0,0)=0$ or $\phi(x,y,0)=0$ when $z=0$

(ii) $\phi \to 0$ far from the charge $(x^2+y^2+z^2 >> d^2)$

Now, forget about actual Problem, and Consider a completely different situation In which we have two point charges and no Conducting plane as fig.(2).

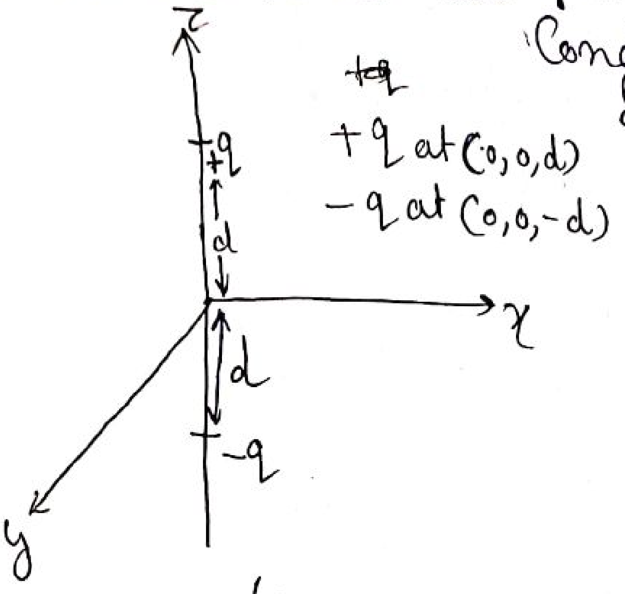


fig.(2)

For this Configuration the potential is given by

$$\phi(x,y,z) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{[x^2+y^2+(z-d)^2]^{1/2}} - \frac{q}{[x^2+y^2+(z+d)^2]^{1/2}}\right] \quad \text{——— (1)}$$

denominators represent the distances from point of charges q and q' (x,y,z).

Now, from eqn (1) it follows that $\boxed{\phi(x,y,0) = 0}$ as $z\to 0$

Satisfies Ist boundary cond^n

and this finds that

$$\phi(x,y,z)\to 0 \quad x^2+y^2+z^2 >> d^2$$

from eq^n (1) it is clear that than only charge in the Region is q, for

$$z>0,\ +q\ (0,0,d)$$

but these are boundary Cond^{ns} for Original prob.

Second configuration produces exactly the same potential as Ist ". in upper Region.

Conclusion is that the potential of a point Charge above an infinite grounded Conductor is given by (for $z \geq 0$) in eq^n (1)

Now, you have to calculate induces surface charge density we know that,

$$\frac{\sigma}{\varepsilon_0}\hat{n} = E = -\nabla\phi$$

$$\frac{\sigma}{\varepsilon_0}\hat{n} = -\nabla\phi$$

$$\frac{\sigma}{\varepsilon_0}\hat{n}\cdot\hat{n} = -\nabla\phi\cdot\hat{n}$$

$$\frac{\sigma}{\varepsilon_0} = -\frac{\partial\phi}{\partial n}$$

then, $\boxed{\sigma = -\varepsilon_0\frac{\partial\phi}{\partial n}}$

In this case,
this is unit normal vector $\hat{n}$, is in z-direction
then

$$\sigma = -\varepsilon_0\frac{\partial\phi}{\partial z}\bigg|_{z=0}$$

(which is induced surface charge density)

from eq^n (1)

$$\frac{\partial\phi}{\partial z} = \frac{1}{4\pi\varepsilon_0}\left[\frac{-q(z-d)}{[x^2+y^2+(z-d)^2]^{3/2}} + \frac{q(z+d)}{[x^2+y^2+(z+d)^2]^{3/2}}\right]$$

then σ becomes

$$\sigma = -\frac{\varepsilon_0}{4\pi\varepsilon_0}\left[\frac{-q(z-d)}{(x^2+y^2+(z-d)^2)^{3/2}} + \frac{q(z+d)}{(x^2+y^2+(z+d)^2)^{3/2}}\right]_{z=0}$$

at $z=0$

$$\boxed{\sigma(x,y) = -\frac{q\,d}{2\pi(x^2+y^2+d^2)^{3/2}}} \qquad (2)$$

as accepted if q is positive then σ is negative is greatest at origin

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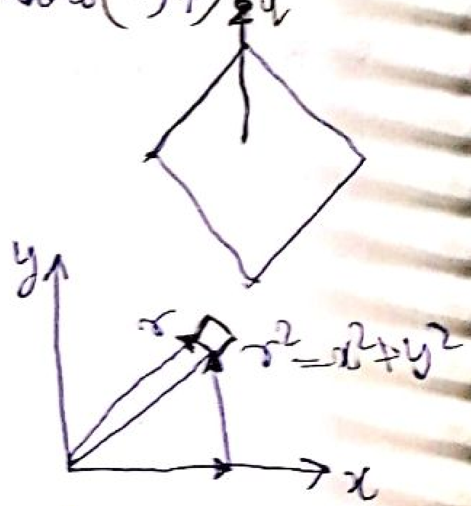
σ_{max} at $x=y=z=0$

$$\sigma(0,0,0) = \cancel{\frac{q}{2\pi d}} \cdot \frac{-q}{2\pi d^2}$$

total induced charge Q is given by,

$$Q = \int \sigma \, da \quad \text{(3) (integral is over } x\text{-}y \text{ plane)}$$

it is simplest to use ~~angle~~ co-ordinates (r, ϕ)



eq^n (2) becomes

$$\sigma(x,y) = \frac{-qd}{2\pi(r^2+d^2)^{3/2}}$$

then by (3)

$$Q = -\frac{qd}{2\pi}\int_{\phi=0}^{2\pi}\int_{r=0}^{\infty} \frac{r\,dr\,d\phi}{(r^2+d^2)^{3/2}}$$

$da = r\,dr\,d\phi$

$$Q = -qd\int_{r=0}^{\infty} \frac{r\,dr}{(r^2+d^2)^{3/2}}$$

$$= -qd \cdot \left[-\frac{1}{(r^2+d^2)}\right]_0^{\infty} = qd \cdot \left[\frac{1}{(r^2+d^2)^{1/2}}\right]_0^{\infty}$$

$$= -q \cdot d \cdot \frac{1}{d}$$

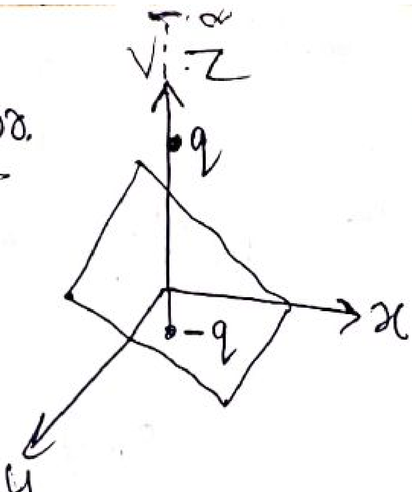
$$\boxed{Q = -q}$$

evedently, the total charge induced on the plane is $-q$.

Force and Energy:- The charge q is attracts towards the plane bcoz of negative ~~&~~ induced charge, Let us calculate the force of attraction. Since, potential in vicinity of q is same as an analogue prob. but no Conductor.

So also the field and Hence force,

$$\underline{F} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{(2d)^2} \cdot \hat{z} \qquad \text{——— (4)}$$



with two point charges and no Conductor energy is given by—

$$W = -\frac{1}{2} \frac{q^2}{4\pi\varepsilon_0 \cdot d}$$

first of all, using, $W = \frac{1}{2} \sum_i q_i V_i$

let, $V_i = \frac{q_i}{4\pi\varepsilon_0 d}$ (due to q_i)

then,

$$W = -\frac{1}{2} \frac{q^2}{4\pi\varepsilon_0 d} \qquad \text{——— (5)}$$

But for a single charge and Conducting plane the energy is half of this:— Since the energy Stored in the fields

then, $W = \frac{\varepsilon_0}{2} \int E^2 d\tau$

$$W = -\frac{q^2}{4\pi\varepsilon_0 (4d)} \qquad \text{——— (6)}$$

In eqn (5) both region $z<0$ (lower) and $z>0$ (upper region) both Contributes. but in eqn (6) only upper region Contains a non zero field. and Hence the energy is half as great.

One Can Energy Can be calculating by work done to bring q in from ∞. the Force required, to oppose is eq^n (4) .. then,

$$\underline{F} = \frac{+1}{4\pi\varepsilon_0} \cdot \frac{q^2}{(2d)^2} \cdot \hat{z}$$

and Work done is

$$W = \int_{\infty}^{d} \underline{F} \cdot \underline{dl}$$

but in z-direction

$$W = \int_{\infty}^{d} \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{(2z)^2} dz$$

$$= \frac{q^2}{4\pi\varepsilon_0} \cdot \frac{1}{4}\left[-\frac{1}{z}\right]_{\infty}^{d}$$

$$\boxed{W = -\frac{q^2}{(4\pi\varepsilon_0) \cdot 4d}}$$

as we move q towards Conductor then we work only on q. Induced charge is moving in over Conductor but it costs nothing, since whole Conductor is at zero potential. In Contrast, If we bring in two point charges with no Conductor, we will do work on both of them, then energy will be twice of that.